-4 m.c.

Test 3A - MTH 1310

Dr. Graham-Squire, Fall 2012

1:21
1:50
29 -4 = 25 m.c.

Name: Key

I pledge that I have neither given nor received any unauthorized assistance on this exam.

_____

(*signature*)

DIRECTIONS

1. Show all of your work and use correct notation. A correct answer with insufficient work or incorrect notation will lose points.
2. Clearly indicate your answer by putting a box around it.
3. Cell phones and computers are <u>not</u> allowed on this test. All cell phones should be turned off and put away, if I see a cell phone out it will be considered an honor code violation.
4. Calculators <u>are</u> allowed on the first 5 questions of the test, however you should still show all of your work. No calculators are allowed on the last 4 questions.
5. Give all answers in exact form, not decimal form (that is, put π instead of 3.1415, $\sqrt{2}$ instead of 1.414, etc) unless otherwise stated.
6. If you need to use the quadratic formula, it is $\dfrac{-b \pm \sqrt{b^2 - 4ac}}{2a}$.
7. Make sure you sign the pledge.
8. Number of questions = 8. Total Points = 80.

1. (12 points) You are given that

$$f(x) = \frac{x}{1+x^2}, \qquad f'(x) = \frac{1-x^2}{(1+x^2)^2}, \qquad \text{and } f''(x) = \frac{-2x(3-x^2)}{(1+x^2)^3}$$

Answer the following questions, and make sure to show your work.

(a) On what interval(s), if any, is f increasing?

(b) On what interval(s), if any, is f concave down?

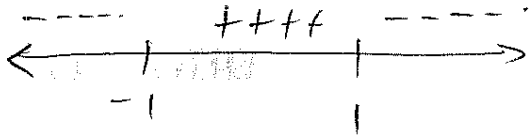
(c) What are the (x, y)-coordinates of the local maximum(s), if any, of f?

(d) What are the (x, y)-coordinates of the inflection point(s), if any, of f?

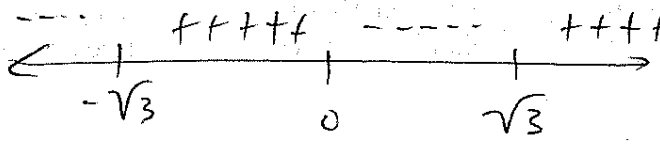
$f'(x) = 0$ if $1 - x^2 = 0 \Rightarrow (1-x)(1+x) = 0$

$\Rightarrow x = 1$ or $x = -1$

$f'(x)$ always exists.



$f''(x) = 0$ at $x = 0$, $x = -\sqrt{3}$ and $x = \sqrt{3}$



(a) $(-1, 1)$

(b) $(-\infty, -\sqrt{3})$ and $(0, \sqrt{3})$

(c) max at $x = 1 \Rightarrow y = f(1) = \frac{1}{2}$

max at $\left(1, \frac{1}{2}\right)$

(d) inflection points at $x = -\sqrt{3}, 0, \sqrt{3}$

$\Rightarrow \left(-\sqrt{3}, \frac{-\sqrt{3}}{4}\right), (0,0), \left(\sqrt{3}, \frac{\sqrt{3}}{4}\right)$

2. (10 points) Use calculus to find the absolute maximum and minimum of

$$f(x) = \frac{2}{3}x^3 - \frac{x^2}{2} - 3x + 2$$

on the interval [-2,4].

1:30
1:33
refigure.

$f'(x) = 2x^2 - x - 3 = (2x - 3)(x + 1)$ ✓✓

$f'(x) = 0$ if $x = \frac{3}{2}$ or $x = -1$ ✓✓

4

$f(-2) = 0.66$

$f(4) = \frac{128}{3} - 8 - 12 + 2 = 24.66$ ← Max

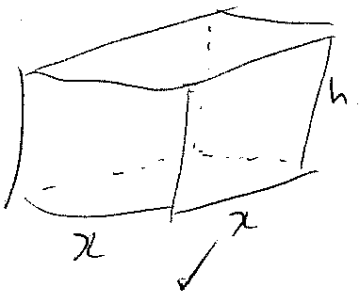
$f(\frac{3}{2}) = \frac{9}{4} - \frac{9}{8} - \frac{9}{2} + 2 = \frac{9}{8} - \frac{5}{2} = -1.375$ ← Min

$f(-1) = \frac{-2}{3} - \frac{1}{2} + 3 + 2 = \frac{7}{3} + \frac{3}{2} = \frac{23}{6} \approx 3.83$

3. (12 points) Eva wants to paint the inside of a rectangular room with a square base. The room must have a volume of 1200 ft^3. The paint for the walls costs \$0.05 per ft^2 and the paint for the ceiling costs \$0.02 per ft^2.

(a) Find the dimensions of the room that will minimize the cost of the paint. Round your answers to the nearest 0.01 ft.

(b) Use either the first derivative test or the second derivative test to confirm that your answer is a minimum.



Ceiling area $= x^2$ ft

wall area $= 4xh$

Volume $= x^2h = 1200$ ✓

$\Rightarrow h = \frac{1200}{x^2}$

Cost $= 0.02x^2 + 0.05(4xh)$ ✓

$C(x) = 0.02x^2 + 0.2x\left(\frac{1200}{x^2}\right)$ ✓

$C(x) = 0.02x^2 + \frac{240}{x}$

$C'(x) = 0.04x - \frac{240}{x^2}$ ✓✓

✓✓ $0 = 0.04x - \frac{240}{x^2}$

$\frac{240}{x^2} = 0.04x \Rightarrow x^3 = \frac{240}{0.04}$ ✓

$= x = \sqrt[3]{6000} \approx 18.17$

(a) $\Rightarrow$ Sides are 18.17 ft, height is $\frac{1200}{(18.17)^2} \approx 3.63$ ✓

(b) ✓ ---- | ++++ $\quad C'(17) = -0.15 \quad C'(20) = 0.2$

18.17 ↖ Min.

4. (8 points) The percentage of families that were married households between 1970 and 2000 is approximately

$$P(t) = 86.9e^{-0.05t} \qquad (0 \leq t \leq 3)$$

where t is measured in decades, with $t = 0$ corresponding to 1970.

(a) What was the percentage of families that were married in 1980? Round to the nearest 0.01.

(b) In what year (approximately, round to the nearest year) did the percent of married households drop to 80.6%?

(a) in 1980, $t = 1$

$\Rightarrow P(1) = 86.9e^{-0.05(1)} = \boxed{82.66\%}$ ✓✓

(b) $80.6 = 86.9e^{-0.05t}$ ✓

$\frac{80.6}{86.9} = e^{-0.05t}$ ✓

$\ln\left(\frac{80.6}{86.9}\right) = \ln e^{-0.05t}$ ✓

$\frac{\ln\left(\frac{80.6}{86.9}\right)}{-0.05} = t$ ✓

$t \approx 1.5 \Rightarrow \approx \boxed{1985}$ ✓

5. (6 points) Adam bought a house in 2012 for \$292,000. Assuming that the worth of the house increases by 2.3% a year, compounded continuously, in what year will the house be worth \$350,000? Round to the nearest year.

$A = Pe^{rt}$ ✓

$P = 292{,}000$

$r = 0.023$

$A = 350{,}000$

Want t.

$350{,}000 = 292{,}000\, e^{0.023t}$ ✓✓

$\frac{350}{292} = e^{0.023t}$ ✓

$\ln\left(\frac{350}{292}\right) = \ln e^{0.023t}$

$\frac{\ln\left(\frac{350}{292}\right)}{0.023} = t$ ✓

$t = 7.877$

$\Rightarrow$ in 8 years, so about $\boxed{2020}$ ✓

-0.5 it don't say 2020

NO CALCULATORS FOR THIS PART

6. (8 points) Use implicit differentiation to find $\frac{dy}{dx}$ for the equation

$$\frac{d}{dx}\left(7x+\frac{x}{y}+y^4=13.\right)$$

$$7+\frac{1\cdot y - x\cdot\frac{dy}{dx}}{y^2}+4y^3\cdot\frac{dy}{dx}=0$$

$$\frac{y}{y^2}-\frac{x}{y^2}\frac{dy}{dx}+4y^3\frac{dy}{dx}=-7$$

$$\left(4y^3-\frac{x}{y^2}\right)\frac{dy}{dx}=-7-\frac{1}{y}$$

$$\frac{dy}{dx}=\boxed{\frac{-7-\frac{1}{y}}{4y^3-\frac{x}{y^2}}}$$

7. (10 points) You are backpacking in the woods when you make a terrible puppy-killing math mistake. It is so bad that a tree falls on your leg, and won't release you until you answer the following questions without the use of a calculator:

The relationship between Weasel Realty's yearly profits, $P(x)$, and the amount of money x spent on advertising per year is given by

$$P(x) = \frac{-1}{8}x^2 + 7x + 30 \qquad (0 \le x \le 50)$$

where both $P(x)$ and x are measured in thousands of dollars.

(a) *Use differentials* to estimate the change in profits when advertising expenditure is increased from \$24,000 to \$26,000.

(b) Is it a good idea to increase the advertising expenditure described above? Explain why or why not.

$dy = f'(x_0)\,dx$ ✓✓ $\quad x_0 = 24$ ✓ $\quad dx = 2$ ✓

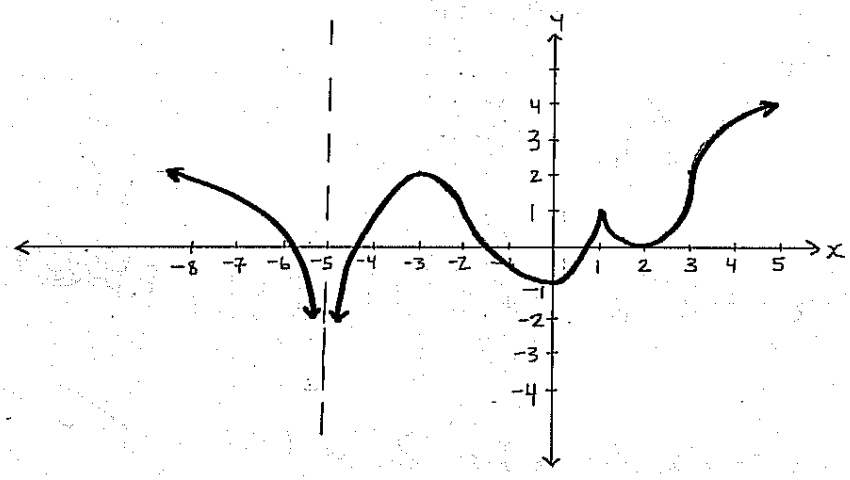
(a) $P'(x) = \frac{-2}{8}x + 7 = \frac{-x}{4} + 7$ ✓

$dy = P'(24) \cdot 2 = \left(\frac{-24}{4} + 7\right)2 = (-6+7)\cdot 2 = 2$ ✓

$\Rightarrow$ profit increase \$2000 ✓

(b) Yes, because profits go up by \$2000 so you are making more money. ✓✓

8. (8 points) Answer the questions for the graph of the function $f(x)$.



(a) On what interval(s), if any, is f decreasing?

$(-\infty, -5)$, $(-3, 0)$, $(1, 2)$ ✓✓

(b) On what interval(s), if any, is f concave up?

$(-2, 1)(1, 3)$ ✓✓

(c) What are the (x, y)-coordinates of the local minimum(s), if any, of f?

$(2, 0)$ and $(0, -1)$ ✓✓

(d) What are the (x, y)-coordinates of the inflection point(s), if any, of f?

$(-2, 1)$, $(3, 2)$ ✓✓

or

$(-3/2, 0)$

9. (6 points) Use logarithm rules to expand and simplify, as much as possible, the expression

$$\ln \frac{x^2}{e^{\sqrt{x}}(1+x)^2}$$

$= \ln x^2 - \ln\left(e^{\sqrt{x}}(1+x)^2\right)$

$= 2\ln x - \left(\ln e^{\sqrt{x}} + \ln (1+x)^2\right)$

$= 2\ln x - \sqrt{x}\,(\ln e) - 2\ln(1+x)$

$= 2\ln x - \sqrt{x} - 2\ln(1+x)$

Extra Credit(2 points)